HelioScope: Mathematical Formulation

Our goal is to develop a complete understanding of solar system performance, and to develop this understanding into an accurate model of solar output. When a new solar project is designed, the nameplate capacity is usually the point of focus, but in reality a number of factors such as temperature, shading, module mismatch, line loss, inversion efficiency, and any number of other performance drivers can greatly influence system performance. All in, these factors account for 20-30% of system performance, but most solar evaluation models pay only cursory attention to them.

HelioScope attempts to incorporate all performance factors on a field, in as transparent a way as possible. As such, the purpose of this documentation is to provide a complete summary of the entire model

Background

HelioScope is designed to simulate solar array output for a specific location under a range of technology, architecture and environmental assumptions. This is accomplished through a component based system model – simulating each electrical component within the solar array individually, and allowing them to interact in a realistic way. This allows the model to compute losses and performance dynamics within a solar field from first principles -- in contract to existing models which use a single module performance model, and assume its output scales directly with field size, adding a series of assumptions about typical loss profiles due to factors such as line loss, mismatch, shading, etc.

Model Components

The **System Model** calculates all the electrical relationships and models field output for a given time under a given set of assumptions. Rather than modeling a single Photovoltaic (PV) Module and assuming all models are identical, the system model includes each PV Module individually and allows them to interact just as they would in reality.

The **Environment Model** provides all the assumptions for the System Model. Modeling a field based on individual components and their geographic location opens up a range of opportunities for the environmental modeling. HelioScope by its nature has robust shading calculations; impaired models accurately affect all other models in their string, and then the system. Additionally, temperature gradients across a field, or dust from nearby roads can impact just the necessary regions of the field. The environmental model behind HelioScope begins with NREL weather data, and incorporates a range of analysis based on each component in the field.

System Model

Each solar array is simulated by directly inputting all the physical components into the system in a hierarchical system duplicating the electrical design. In this system, the inverter sets the ultimate draw voltage for the entire field, this voltage is communicated through all the components (wires, combiners, power optimizers) until it reaches individual modules; these modules then respond by supplying current through the system according to environmental factors. Each components behavior is guided by their underlying physical equations or behavior rules (for power optimizers), and the model ensures that all voltages and currents within the system satisfy these equations.

Inverters

The inverter is at the root of the model, setting the initial draw voltage that propagates through all components connected in the array. The inverter has two core functions in the model AC inversion, and maximum power point tracking (MPPT). For arrays using distributed electronics, the behavior of MPPT can and does vary.

AC Inversion

To simulate AC inversion, total DC field output (voltage and current) are subject to the inversion efficiency curves as published by the California Energy Commission (CEC). These curves are inverter specific and provide efficiency for three voltages and five power levels.

To calculate specific inversion efficiency, we use a simple linear interpolation between the nearest adjacent points. That is to say:

1. For the nearest voltage level above and below the inverter DC voltage, extrapolate the efficiency based on the current DC power
2. Extrapolate the efficiency at the actual DC voltage level using the voltage and efficiency points determined in (1).

Mathematically, to find the efficiency at a given power and voltage ($\eta_{v,p}$) we use simple linear extrapolation. For example, if p_1 is the nearest power point below the target power, and p_2 is the nearest point above the target power, the extrapolated efficiency at a given voltage is given by:

$$\eta_{v,p_1} = \eta_{v_{min},p_1} + (v - v_1)\left(\frac{\eta_{v_2,p_1} - \eta_{v_1,p_1}}{v_2 - v_1}\right)$$

Similarly for p_2:

$$\eta_{v,p_2} = \eta_{v_{min},p_2} + (v - v_1)\left(\frac{\eta_{v_2,p_2} - \eta_{v_1,p_2}}{v_2 - v_1}\right)$$

This gives us a total extrapolated efficiency of:

$$\eta_{v,p} = \eta_{v,p_1} + (p - p_1)\left(\frac{\eta_{v,p_2} - \eta_{v,p_1}}{p_2 - p_1}\right) \tag{1}$$

Note that this approach requires points to be a on a 'grid' for the algorithm to work – efficiency values for multiple power points must exist for each voltage level.

Maximum Power Point Tracking (MPPT)

When an array does not utilize any form of distributed MPPT or other power optimization, we simulate this functionality by having the inverter continuously adjust the draw voltage level until a peak power point is reached (just as an inverter does in reality). However, this algorithm is idealized in that is assumes perfect tracking (although subject to local extrema) – there are no time-lag penalties associated with changing insolation or other conditions.

To find the MPP, the inverter simply scans through the range of potential draw voltages, synchronizing the voltages across the system at each point, and does not stop until it has found a local extrema (with exit criteria in terms of both dP/dV as well as in terms of absolute convergence.

Performance Limits

The inverter is fixed to perform within its operating characteristics, the modeled performance constraints are:

- An inverter cannot operate above its maximum operating voltage, or below its minimum operating voltage
- If an inverter cannot operate above its maximum power, any output above the maximum power is clipped (on the DC side)

Combiners

A combiner simply connects several wires in parallel, aggregating current. It takes any number of input wires, ensures that they are all outputting at the same voltage (whatever the draw voltage of the output line is) and then outputs the total current to a single output wire. Combiners are assumed to have 100% transmission efficiency.

Definition of a parallel connection for the combiner:

$$\text{for each input, } i : V_{combiner} = V_i \tag{2}$$

Perfect efficiency at aggregating current:

$$I_{combiner} = \sum_{inputs} I_i \tag{3}$$

Wires

Wires are the fundamental piece connecting all components in the system – in this model every wire propagates current while subjecting the source (input) voltage to a voltage drop according to resistance across the conductor.

$$V_{load} = V_{source} - I \cdot (2L\rho) \tag{4}$$

Here, L is the length of the wire, and ρ is the published resistivity of the wire gauge.

$$I = I_{out} = I_{in} \tag{5}$$

While the above equation describes the voltage drop for the connection of two simple components at either end of a wire; when several components are in parallel or series along the same lines the relationships are slightly more complex.

Series Connection

When components are connect in series, voltage accumulates along the string, while current is constant.

To model this, all components (modules and power optimizers) connected to a series string are constrained to operate at the same current. Given this constraint, the output voltage for a series string V_{load} is the sum of the individual voltage contributions (v_i) of each module, less the total line losses based on the current, I, on the string (where L is the total string length).

$$V_{load} = \sum_{i=1}^{n} v_i - I \cdot (2L\rho) \tag{6}$$

Parallel Connections

When components are connected in parallel, current accumulates along the bus, while voltage is effectively constant. The load (output) end of the bus carries the full current of all the modules, while the tail end has no current. The means that the draw voltage at a given point depends on the current and resistivity between that point and the load.

When operating on a parallel bus, components are constrained to output at the location specific draw voltage for that device - this is directly analogous to a combiner, the only difference is that a parallel bus also has a slight rise in draw voltage from device to device due to line losses. This voltage V_i for each component, i, can be calculated based on the voltage of the next nearest component (V_{i-1}), the distance between those two components ($L_i - L_{i-1}$) and total current on that segment (the sum of all the current that has accumulated on the bus thus far, $\sum_{j=i}^{n} I_j$).

This is given by:

$$V_i = V_{i-1} + 2(L_i - L_{i-1})\rho \sum_{j=i}^{n} I_j \tag{7}$$

In this equation, the load voltage at the end of the line is given by V_0, and has a position $L_0 = 0$. As in combiners, the current is the sum across all currents on the bus:

$$I_{bus} = \sum_{components} I_i \tag{8}$$

Distributed Electronics

Distributed electronics are treated by HelioScope as any other item in the field, however, they often have more complicated behavior rules. Typically, they will provide their own MPPT for the localized modules (or strings) underneath them, and will then present to the inverter (or other connected components) a relatively stable power curve, that is only impacted by their efficiency across the range of voltage and current output.

Tigo Maximizers

When modeling arrays using a Tigo architecture, a Module Maximizer is placed between each module and it's parent string (wired in series). This Maximizer optimizes the module power output individually and then contributes the appropriately 'impedance matched' current and voltage to the rest of the system. Effectively, the Maximizer performs an I/V transformation to each module, allowing modules with currents that vary significantly from that of the rest of the string to contribute at maximum power.

HelioScope models Tigo Maximizer using the buck optimizer algorithm described below and published efficiency curves.

eIQ vBoosts

When modeling arrays using an eIQ architecture, the inverter runs at a constant voltage, and every module is wired in parallel. Each vBoost is individually responsible for outputting a voltage high enough to match the array operating point.

HelioScope models eIQ vBoosts using the boost optimizer, with a fixed inverter draw voltage of 400V (the current design recommendation).

Buck Optimizers

Buck optimizers operate between a module (or modules) and it's corresponding bus (a series string). This optimizer is a DC/DC converter that transforms the module output voltage and current allowing a module to be driven at an operating power point (voltage and current) that is independent of the other modules in the array.

A buck optimizer is constrained in that it can only buck, or reduce, the module output voltage (which increases the effective current output). This means if the array operating point for a given module is below the module's maximum power voltage (V_{mp}), or equivalently above the modules maximum power current (I_{mp}, the module can still operate at or near it's peak output. System level algorithms can also adjust the power transformation for all the optimizers as a group to perform more global optimizations.

The buck optimizer's performance benefits do incur performance penalty, primarily driven by the magnitude of the voltage transformation - larger voltage transformations typically incur a larger performance penalty.

Efficiency Calculations

Optimizer efficiency is calculated based on the *bucking ratio*, χ, which is defined to be:

$$\chi = \frac{V_{out}}{V_{in}} \tag{9}$$

Where V_{out} is the effective output voltage (given the Tigo impedance correction), and V_{in} is the maximum power point voltage (V_{mpp}) of the attached module.

Extrapolating Efficiency from a Curve

Based on the selected bucking ratio, efficiency is then extrapolated from a table of actual values, using a linear extrapolation

$$\eta_\chi = \eta_{\chi_{min}} + (\chi_{max} - \chi_{min}) \left(\frac{\eta_{\chi_{max}} - \eta_{\chi_{min}}}{\chi_{max} - \chi_{min}} \right) \tag{10}$$

Output Transformation

Given equation (9), the transformation for current, in response to voltage bucking is defined to be:

$$I_{out} = \frac{I_{in}}{\chi} \cdot \eta(\chi) \tag{11}$$

Which means that output power is given by:

$$P_{out} = \left(\frac{I_{in}}{\chi} \cdot \eta(\chi) \right) \cdot \left(V_{in} \cdot \chi \right) = I_{in} V_{in} \eta(\chi) \tag{12}$$

Field Dynamics

Oftentimes buck optimizers are governed by system-level algorithms to ensure that modules are individually operating at their maximum power points (MPPs) by setting χ in advance based on field conditions. In practice these algorithms are typically proprietary so HelioScope models this behavior by performing a heuristic search for the MPP available to the inverter given the presence of the Maximizer units.

This search assumes that each optimizer's behavior is governed by two rules:

1. Increase χ to change the I/V curve when the array operating draw voltage is lower than the module MPP
2. Enter bypass mode ($\chi = 1$) when the inverter draw voltage is above the module MPP

This heuristic search will match field performance given the following assumptions:

- The system algorithm correctly identifies the maximum power point for every module in the array
- The system algorithm internalizes and optimzier losses
- The power loss of the optimizer I/V transformation (when reducing voltage) is smaller than the module power loss (from P_{mp}) when reducing the draw voltage (because the optimizers will always choose to buck voltage in this case)

Boost Optimizers

A boost optimizer is a DC/DC converter that when placed on an array optimizes the power output of any connected modules and then boosts their voltage up to a level high enough to match the array operating point. Just as with buck optimizers, boost optimizers allow any connected to modules to operate independently from the rest of the system.

The primary difference between boost optimizers and buck optimizers is that boost optimizers operate in a parallel system – output constrained to match the array voltage, and current adds independently (as opposed to a series string where current must be the same and voltage adds independently). This means that when using boost optimizers, the entire array must be wired in parallel, and the inverter operates at a fixed draw voltage.

Given that the inverter operates at a fixed draw voltage, and that the optimizers are connected on a parallel bus, every optimizer must output at a voltage given by equation (7). HelioScope assumes that the boost optimizer correctly tracks the module (or group of modules) maximum power point correctly and supplies the appropriate voltage and current to the array.

The voltage boost incurs a performance cost that is driven by both input power and voltage. The efficiency of each boost optimizer is governed by equation (1), the same extrapolation method used for inverters.

Module Model

The Module Model is the core of any PV System Performance Model, and is the most significant driver of the overall level of field performance. The Module Model defines the electrical behavior of a photovoltaic module given the specific irradiance and temperature of that module.

There are several commonly used module models, most are based on the full-diode equation (e.g.the PVSyst Model), some newer models are based on real world performance (e.g. the Wisconsin Five-Parameter model). Currently, we offer models based on the PVSyst Model (that can be imported from .PAN files), as well as a characterization based on the full-diode model. As more accurate performance models are developed for new modules, we will update our algorithms to incorporate these newer performance models.

Single-Diode Model

The Single-Diode Model is a characterization based around the full-diode equation, with key adjustments to ensure module behavior is accurate across a range of cell-temperatures.

The single diode equation is given by:

$$I = I_{sc} \cdot S - I_0(T)\left(e^{\frac{q}{kTa}(V+IR_s)} - 1\right) - \frac{V + IR_s}{R_p} \tag{13}$$

These parameters can be derived from the published module specifications, leaving only environmental variables (T and S). Typically to solve for this equation we assume the short circuit current is correct, and try to fit appropriate parameters for R_s, R_p, I_0 and a based on the published data points (maximum power voltage and current, short-circuit current, and open-circuit voltage).

Temperature Adjustment

Temperature derating plays a substantial role in module performance: increases in cell temperature have a strong impact on open-circuit voltage (typically -0.37%$^\circ C$) for crystalline modules) and a minimal impact on current (0.05%$^\circ C$) resulting in a net power impact of -0.5%$^\circ C$). For each module, we calculate a temperature adjustment factor, τ, to the saturation current to provide the desired relationship.

$$I_0(T) = I_0(1 + \tau)^{T-25} \tag{14}$$

This parameter ignores changes to short circuit current, but ensures that the voltage and power coefficients are consistent with published module specifications for the common range of cell temperatures.

Bypass Diodes

Typically modules have 2-3 bypass diodes. In this model we simulate each module as having only one 'perfect' bypass diode – if ever a module cannot produce at the voltage or current required of it, the module has no impact on the system, current passes directly through it.

PVSyst Diode Model

The PVSyst Module Model, developed by Andre Mermud, is a modified single-diode based model, that leverages Andre's experience and research in the field to accurately parameterize Photovoltaic Models based on known physical characteristics of the technology (e.g. Crystalline vs. Thin-film modules) and the available specifications.

At its core the PVSyst Module Model has two core pieces, the actual formulation, which includes many adjustments to the standard Shockley Full-Diode Equation, and a careful system for parameterizing each module.

Mathematical Formulation

The Shockley Single-Diode Model can be written as:

$$I = I_{ph} - I_0\left(e^{\frac{q}{kT}\cdot\frac{1}{\gamma N_{cs}}(V+IR_s)} - 1\right) - \frac{V + IR_s}{R_{sh}} \tag{15}$$

The PVSyst module model is based around this fundamental diode formula, but parameterizes several of the factors (e.g. I_{PH}), as can be seen in the table below.

Module Output	I	Current supplied by the module
	V	Voltage supplied by the module
Inputs	G	Effective Irradiance W/M^2
	T_C	Module (Cell) Temperature, K
Module Parameters	$I_{PH}(G,T)$	Photocurrent (e.g. Short-Circuit Current), proportional to G, (Amps)
	$I_0(T)$	inverse saturation current, depending on the temperature, (Amps)
	$R_{sh}(G)$	Shunt Resistance, varies with G, (Ohms)
	R_s	Series Resistance, (Ohms)
	γ	Module quality factor (typically 1-2)
	N_{CS}	

		Number of cells in series the module
Physical Constants	q	Fundamental charge of the electron, $1.602 \cdot 10^{-19}$ Coulomb
	K	Boltzmann constant, $1.381 \cdot 10^{-23}$ J/K)

Photocurrent Calculation

As can be seen above, the photocurrent, or short circuit current, determines the maximum current the module can output for a given level of irradiation (G) and at a given temperature (T). The precise formula in the PVSyst documentation is:

$$I_{PH}(G, T_c) = \left(\frac{G}{G_{ref}}\right) \cdot \left(I_{PHref} + \mu_{Isc}(T_C - T_{Cref})\right) \tag{16}$$

Output	I_{PH}	Short-Circuit Current of the Module
Inputs	G	Effective Irradiance, W/M^2
	T_C	Module (Cell) Temperature, K
Module Parameters	I_{PHref}	Reference Photocurrent, e.g. published short-circuit current (amps)
	μ_{Isc}	Temperature Coefficient of the Photocurrent, A/K
Reference Inputs	G_{ref}	Reference Irradiance (typically 1000 W/M^2)
	T_{ref}	Reference Temperature (typically 25 °C)

Reverse Saturation Current

The reverse saturation current determines what voltage the module can provide for any given level of irradiance and current. It depends on the current cell temperature (T_C) and module parameters.

$$I_0(T_C) = I_{0ref}\left(\frac{T_C}{T_{Cref}}\right)^3 \cdot e^{\left(\left(\frac{q \cdot E_{gap}}{\gamma \cdot k}\right)\left(\frac{1}{T_{Cref}} - \frac{1}{T_C}\right)\right)} \tag{17}$$

Output	I_0	Diode Saturation Current (amps)

Inputs	T_C	Module (Cell) Temperature, K
Module Parameters	I_{0ref}	Reference saturation current for the module, amps
	γ	Module quality factor (typically 1-2)
	E_{gap}	Bandgap for the semiconductor, see table, eV
Reference Inputs	T_{ref}	Reference Temperature (typically 25 $^\circ C$), K
Physical Constants	q	Fundamental charge of the electron, ($1.602 \cdot 10^{-19}$ Coulomb)
	K	Boltzmann constant, ($1.381 \cdot 10^{-23}$ J/K)

Shunt Resistance Calculation

The PVSyst model includes an insolation dependence for the shunt resistance R_{sh}, this is typically expected for a-Si modules, but has been observed for all modules, in fact, all technologies have demonstrated a similar dependence on insolation, as seen below>

$$R_{SH}(G) = R_{SH}(G_{ref}) + \left(R_{SH}(0) - R_{SH}(G_{ref})\right) \cdot e^{\left(-\beta \frac{G}{G_{ref}}\right)} \tag{18}$$

Output	R_{SH}	the effective shunt-resistance of the module, Ohms
Inputs	G	Effective Irradiance, W/M^2
Module Parameters	$R_{SH}(G_{ref}$	Shunt resistance at reference Photocurrent, ohms
	$R_{SH}(0)$	Default shunt resistance value
Reference Inputs	G_{ref}	Reference Irradiance (typically 1000 W/M^2)
	β	Experimentally determined decay parameters, typically 5.5

Module Characterization

Given the formulation for a module as given in equation (15), and the associated adjustments for I_{ph} and I_0, we have six parameters that must be characterized for each module, given below:

Required Parameters	I_{0ref}	Reference saturation current for the module, amps
	γ	Module quality factor (typically 1-2)
	E_{gap}	Bandgap for the semiconductor, see table, eV
	R_s	Series Resistance, (Ohms)
	R_{sh}	Shunt Resistance, (Ohms)

PVSyst addresses these unknown variables by using the measured physical datapoints (maximum power, short-circuit current, and open-circuit voltage) to create additional equations:

From the Open-Circuit Voltage Equation, we have $I = 0, V = V_{oc}, G = G_{ref}$, and $T = T_{ref}$. Plugging this into the above equations yields a formulation for shunt resistance, as a function of γ:

$$I_0(T_{ref}) = I_{0ref} \qquad (equation(17))$$

$$I_{PH}(G_{ref}, T_{ref}) = I_{PHref} \qquad (equation(16))$$

$$0 = I_{PHref} - I_{0ref}\left(e^{\frac{q}{KT}\cdot\frac{V_{oc}}{\gamma N_{cs}}} - 1\right) - \frac{V_{oc}}{R_{sh}} \qquad (equation(15), V_{oc})$$

$$I_{sc} = I_{PHref} - I_{0ref}\left(e^{\frac{q}{KT}\cdot\frac{I_{sc}R_s}{\gamma N_{cs}}} - 1\right) - \frac{I_{sc}R_S}{R_{sh}} \qquad (equation(15), I_{sc})$$

$$I_{mp} = I_{PHref} - I_{0ref}\left(e^{\frac{q}{KT}\cdot\frac{vmp+Imp R_s}{\gamma N_{cs}}} - 1\right) - \frac{V_{mp} + I_{sc}R_s}{R_{sh}} \qquad (equation(15), V_{mp}, I_{mp})$$

At this point, there are several ways to solve for the unknown parameters. If we take the semiconductor bandgap (E_{gap}) as a given, we currently have 4 unknown variables and three equations. There are two ways to algebraically compute the unknown parameters by adding a fourth equation:

1. Use the temperature variance in voltage or power
2. Use the derivative of power at around the maximum power point $\frac{dP_{mp}}{dV} := 0$

Citing concerns over whether these methods result in valid physical parameters, PVSyst instead chooses a reasonable value of γ to choose the best values for R_{sh}, R_s, and I_{0ref}. The user is allowed to change the value of R_s with the warning to ensure that the diode quality factor, γ should remain within physical bounds (typically 1-2).

Semiconductor Bandgap

The simplest parameter to determine is the electrical band gap of the semiconductor. This is typically determined as a default from a lookup table given below:

Band-gap (E_{gap}) lookup table:

Crystalline	1.12 eV
CIS	1.03 eV
a-Si	1.7 eV
CdTe	1.5 eV

Gamma (γ) look up table:

Si-Mono	1.3
Si-Poly	1.35
a-Si:H	1.4
a-Si:H tandem	2.8
a-Si:H triple	4.2(measured)
CdTe	1.5(unknown)
CIS	1.5(measured)
AsGa	1.3(unknown)

Temperature Model

Once irradiance for each module has been calculated, we can must calculate the cell temperature so that I/V curves are appropriately adjusted during the simulation process. Currently two approaches are offered, the recommended model was developed by Sandia National Labs and includes an exponential factor for wind effects (increases in wind speed have smaller effects the faster the wind is going). We also include a simpler diffusion model (as is implemented by PVSyst), which simply includes components for constant and wind based temperature diffusion.

Sandia National Labs Model

Once irradiance has been calculated for each module, we must calculate the cell temperature according to the performance Model given by Sandia National Labs Performance Model (D. L. King, W. E. Boyson, J. A. Kratochvil, (2004))

$$T_m = E \cdot \left(e^{a+b \cdot WS}\right) + T_a \tag{19}$$

where:

T_M
Module temperature, °C
T_A
Ambient air temperature, °C
WS
Wind Speed, M/S
E
Solar Irradiance incident on module surface, W/m^2
a
empirically determined coefficient for the upper limit of module temperature at low wind speed
b
empirically determined coefficient for the rate at which module temperature drops with wind speed

For most modules, there is a difference between module temperature (T_M) and cell temperature (T_C), this difference can be accounted for using:

$$T_C = T_M + \frac{E}{E_0} \cdot \Delta T \tag{20}$$

where:

T_C
Cell temperature, °C;
E_0
Reference solar irradiance on module, (1000 W/m^2)
WS
Wind Speed, M/S
ΔT
The temperature difference between module and cell at E_0

MODULE TYPE	MOUNT	A	B	ΔT
Glass/cell/glass	Open rack	-3.47	-0.0594	3
Glass/cell/glass	Close roof mount	-2.98	-0.0471	1
Glass/cell/polymer sheet	Open rack	-3.56	-0.0750	3
Glass/cell/polymer sheet	Insulated back	-2.81	-0.0455	0
Polymer/thin-film/steel	Open Rack	-3.58	-0.113	3

Currently HelioScope defaults to using the values for *Glass/Cell/Polymer* with an Open Rack for any modules on racks, and to using the values for *Glass/Cell/Polymer* with an insulated back for Flush mounts.

Diffusion Model

In this model, cell temperature is determined based on the total energy absorbed by the module (impacted by the total incident irradiance on the module and the module efficiency), less effects for constant temperature diffusion and diffusion that increases base on wind speed. This is the model commonly used in software such as PVSyst.

T_M
Module temperature, °C;
T_A
Ambient air temperature, °C;
U_C
Constant Thermal loss coefficient, W/M^2·°C;
U_W
Windspeed Thermal loss coefficient, W/M^2·°C; / (m/s)
WS
Wind Speed, M/S
α
Absorption coefficient, typically 90%
η_M
Module Efficiency, %

$$U(WS) = U_C + WS \cdot U_W$$
$$U(WS) \cdot (T_M - T_A) = \alpha E(1 - \eta_M)$$

Solving this for module temperature yields:

$$T_M = \frac{\alpha E}{U}(1 - \eta_M) + T_A \tag{21}$$

When using the diffuse model, HelioScope defaults to the values given in the following table, which typically align with PVSyst.

MOUNT TYPE	U_C	U_W
Flush	29	0
Racks	14	0

Environment Model

While the Electrical Component model calculates all the electrical relationships and models field output for a given time under a given set of assumptions, the environmental model is necessary to generate the assumption set for each individual slice of time. The environmental model goes through a series of submodules to calculate the conditions for calculating field output.

1. Meteorological Data, the raw source for Solar Resource and Temperature Information for the Installation
2. Irradiance Calculation, determination of light levels actually available to the collector based on collector and solar angles (i.e. the transposition model)

3. Shading Models, adjustments to light based on external shading objects and cross-shed shading
4. Reflection, adjustments to available irradiance based on the incidence angle of the sunlight
5. Temperature Modeling, Adjustments to cell temperature based on irradiance and ambient weather
6. Other Mismatch, Other mismatch factors that may affect field output can be incorporated on an ad hoc bases

Only after all these environmental factors are calculated and loaded into the field can field output for that hour be determined as described previously in the component model sections.

Meteorological Data

The primary data source for modeling is a meteorological file including irradiance, ambient temperature and wind data. For our modeling we import data from either the NREL National Solar Radiation Database (in TMY3 format) or from the US Department of Energy Plus Data, (in EPW format).

These databases provide both measured and modeled direct normal and global diffuse solar radiation, as well as ambient temperatures on an hourly basis for a Typical Meteorological Year (8760 hours). These data points, in conjunction with the detailed location and orientation of the solar modules allows for an accurate estimation of the raw solar capacity of a field.

For each hour, the following parameters are used for the purposes of our calculation, as these parameters are averaged over the course of an hour, we use the midpoint of each hour as the sample time (important when calculating the solar angles).

I_{GH}
Total Global Horizontal irradiance at Earth's Surface (W/M^2)
I_{DH}
Diffuse Horizontal irradiance at Earth's Surface (W/M^2)
T
The ambient (dry-bulb) temperature (°C)
v
Wind speed (M/S)

The TMY3 files also have many other parameters (e.g. albedo, relative humidity, precipitation, visibility), but many of these are inaccurate or vary in their measurement (or calculation) methodologies from site-to-site, and so are not used.

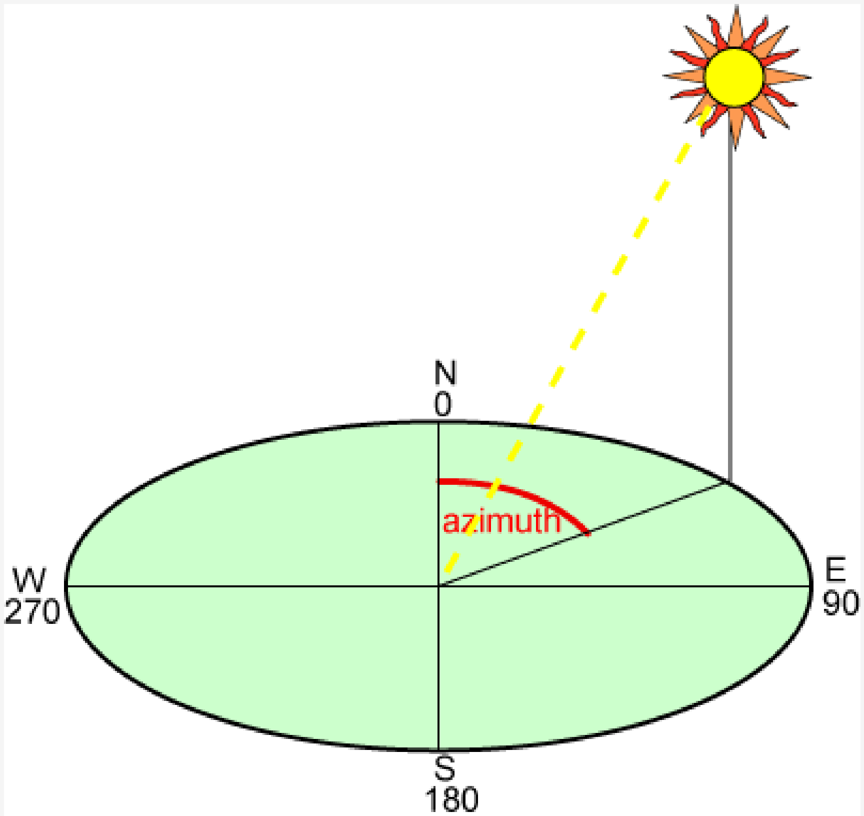
Solar Angles

To incorporate the meteorological data, we must calculate the position of the sun for the project location, at each time in the meteorological file. This calculation depends primarily on the location (latitude and longitude) of the project, and specific time of each measurement. HelioScope uses the PSA algorithm, as developed by Blanco-Muriel et al which is sufficient to calculate the position of the sun to within half an arcminute based strictly on latitude, longitude and time. More detailed calculations require taking elevation, air temperature and air pressure into effect.

For use in verifying results, the NOAA has published an solar position calculator that can be used in Excel.

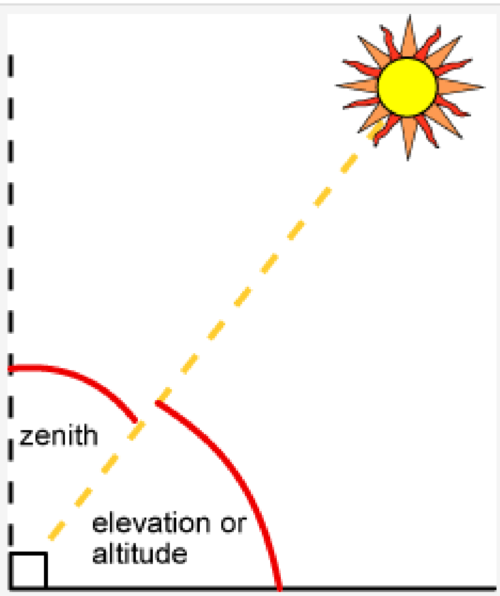
The result of any solar position calculation results in three core values describing the position of the sun relative to the location of the project

ϕ_S
Solar azimuth angle
β_S
Solar altitude angle
Z_S
Solar zenith angle ($90° - \beta_s$)



Solar Azimuth

source



Solar Altitude

source

Irradiance Calculations

To calculate the irradiance available for modules in the system requires calculating the position of the sun for each datapoint in the meteorological data, and then using a tranposition model to convert the information in the meteorological data (Global Horizontal Irradiance and Diffuse Horizontal Irradiance), the solar position (solar azimuth and elevation) and the collector angles (collector azimith and tilt) into available Direct, Diffuse, and Reflected Irradiance

Beam (Direct) Irradiation

The beam irradiation available to the modules is total direct irradiation normal to the surface of the collector. The solar angles necessary for this calculation can be determined for any given time of the a year, as given by the equations below.

I_B
Total Beam Irradiance at the earths surface (W/M^2

I_{BH}
Beam Irradiance incident on a horizontal plane (W/M^2)

I_{BC}
Beam Irradiance incident on the Collector (W/M^2)

θ
Solar incidence angle (to collector)

Σ_C
Collector Tilt

ϕ_C
Collector azimuth angle

We also have several formula for calculating the relevant solar angles:

$$\begin{aligned} cos(\theta) &= cos(\beta_S)cos(\phi_S - \phi_C)sin(\Sigma_C) + sin(\beta_S)cos(\Sigma_C) \\ a &= max\big(0, cos(\theta)\big) \\ b &= max\big(0.087, sin(\beta_S)\big) \end{aligned}$$

The horizontal beam irradiation can simply be calculated as the difference between the measured global and diffuse horizontal irradiation, and the total beam irradiance calculated based on the geometry of the solar angles:

$$\begin{aligned} I_{BH} &= I_{GH} - I_{DH} \\ I_B &= I_{BH} \cdot \frac{1}{b} \end{aligned}$$

From this, the beam component of the radiation incident to the plane of array is given by:

$$I_{BC} = I_{BH} \cdot \frac{a}{b} \tag{22}$$

Diffuse Irradiation

There are two commonly accepted models for calculating the diffuse irradiance available to a collector: the Hays Model and the Perez Model. The Hay's Model is slightly simpler, with only a circumsolar modification to a simple isotropic (sky uniform in all directions) model; the Perez model adds both a circumsolar and horizon coefficient and relies on a table of regression values to develop a more refined model.

The Hay Model for Diffuse Irradiance

The Hay Model depends on the solar constant, I_0, which indicates the amount of solar irradiance available at the top of the Eath's atmosphere to compute a clearness index. This clearness index is then used to compute the circumsolar portion of the diffuse irradiance

I_0
Solar irradiance available outside the Earth's atmosphere (W/M^2)
$\bar{I_0}$
Solar Constant, 1,367.7W/M^2
n
The day number (1..365)
K
The clearness index

$$I_0 = \bar{I_0}\left[1 + 0.033cos\left((n-2)\cdot\frac{360}{365}\right)\right]$$

Note that there is a slight (3.3%) variation in the irradiance available to Earth over the course of the year due to the elliptical nature of Earth's orbit.

Using this irradiance, the Hay model requires the calculation of a clearness index, K given by:

$$K = \frac{I_{BH}}{b \cdot I_0} \tag{23}$$

Combining the above equations we have equation (24) which describes the diffuse irradiance available to a tilted PV collector as given by the Hay Model.

$$I_{DHC} = I_{DH}\left((1-K)\left(\frac{1+cos(\Sigma_C)}{2}\right) + K\,\frac{a}{b}\right) \tag{24}$$

The Perez Model for Diffuse Irradiance

The Perez Model includes both a circumsolar and horizon band coefficient to estimate diffuse irradiation available to the collector.

ϵ
Clearness index
κ
Constant, 1.041
Δ
Brightness Index

m
Air Mass Ratio
F_1
Circumsolar Brightening Coefficient
F_2
Horizon Brightening Coefficient

As in the Hay Model, we must calculate the irradiation available outside the Earth's atmosphere:

I_0
Solar irradiance available outside the Earth's atmosphere (W/M^2)
$\bar{I_0}$
Solar Constant, 1,367.7W/M^2
n
The day number (1..365)

$$I_0 = \bar{I_0}\left[1 + 0.033cos\left((n - 2) \cdot \frac{360}{365}\right)\right]$$

Using this we can calculate the clearness index and brightening coefficient:

$$\epsilon = \frac{\frac{I_{DH} + I_B}{I_{DH}} + \kappa Z_S^3}{1 + \kappa Z_S^3}$$

$$\Delta = \frac{I_{DH} \cdot m}{I_0}$$

Where m, the air mass ratio can be estimated using a method developed by Pickering using the following formulation:

$$m = \frac{1}{sin\left(\beta_S + 244/(165 + 47\beta_S^{1.1}\right)}$$

ϵ BIN	MIN ϵ	MAX ϵ
1 (Overcast)	1.000	1.065
2	1.065	1.230
3	1.230	1.500
4	1.500	1.950
5	1.950	2.800
6	2.800	4.500
7	4.500	6.200
8 (Clear)	6.200	∞

ϵ BIN	F11	F12	F13	F21	F22	F23

1	-0.008	0.588	-0.062	-0.06	0.072	-0.022
2	0.13	0.683	-0.151	-0.019	0.066	-0.029
3	0.33	0.487	-0.221	0.055	-0.064	-0.026
4	0.568	0.187	-0.295	0.109	-0.152	-0.014
5	0.873	-0.392	-0.362	0.226	-0.462	0.001
6	1.132	-1.237	-0.412	0.288	-0.823	0.056
7	1.06	-1.6	-0.359	0.264	-1.127	0.131
8	0.678	-0.327	-0.25	0.156	-1.377	0.251

From values in this table for a given value of epsilon, we can calculate the circumsolar and horizon brightening coefficients:

$$F_1(\epsilon) = F_{11}(\epsilon) + F_{12}(\epsilon) \cdot \Delta + F_{13}(\epsilon) \cdot Z_S$$
$$F_2(\epsilon) = F_{21}(\epsilon) + F_{22}(\epsilon) \cdot \Delta + F_{23}(\epsilon) \cdot Z_S$$

Using these calculated factors for circumsolar and horizon adjustments, we have equation (25), the value for Diffuse Irradiation incident on a collector under the Perez Model.

$$I_{DHC} = I_{DH}\left((1 - F_1)\left(\frac{1 + cos(\Sigma_C)}{2}\right) + F_1 \frac{a}{b} + F_2 sin(\Sigma_S)\right) \tag{25}$$

Reflected Irradiation

Reflected irradiance available to a collector is measured based on the albedo coefficient (the ratio of reflected irradiance to incident irradiance), and the share of the ground the collector can 'see' in front of it. This calculation assumes no near shading objects blocking the share

I_{RC}
Reflected Irradiation in the plane of the collector (W/M^2)
α
Albedo coefficient, typically 0.2

$$I_{RC} = I_{GH} \cdot \alpha\left(\frac{1 - cos(\Sigma_C)}{2}\right) \tag{26}$$

Irradiance Adjustments

After the components of irradiance have been calculated for a single tilted plane, diffuse and reflected irradiance must be adjusted for shading, and reflection due to the incidence angle of incoming irradiation as well as any cross-shed shading. Because the reflection adjustment depends on the location of the incoming light, these two calculations are interdependent and should be performed simultaneously.

Isotropic Adjustment

As was calculated in both the Hay's and Perez model, the reflected and diffuse irradiance available to a module is based on the on the portion of the sky that module can 'see'.

This is calculated by assuming that the sky is isotropic (uniform), so the total diffuse radiation available to any given collector is equivalent to the share of the sky available to it, while adjusting for the portion of the irradiance normal to the surface of the collector (that is the portion pointed directly 'into' the surface of the collector). This fraction can be calculated using a spherical integral - integrating over the surface of a sphere representing the bounds of the sky available to the collector.

$$\Omega = \frac{1}{2\pi} \int_0^{\pi} \int_0^{\pi} sin(\phi)\mathrm{d}\phi\mathrm{d}\theta \tag{27}$$

In this formulation, ϕ is the local azimuth angle, the angle sweeping from east-to-west in front of the collector and θ is the local zenith angle, the angle sweeping over the top of the collector.

Both the Hay and Perez transposition models assume a single bank of modules at a fixed tilt (Σ_C), the fraction of diffuse light, D_{iso} ,available to the modules to be directly proportional to the share of the sky the collector can see.

$$D^0_{iso} = \frac{1}{2\pi} \int_0^{\pi-\Sigma_C} \int_0^{\pi} sin(\phi) \cdot \underbrace{sin(\phi) \cdot sin(\theta + \Sigma_C)}_{\text{share of light 'normal' to collector}} \mathrm{d}\phi\mathrm{d}\theta = \frac{1 + \cos(\Sigma_C)}{2} \tag{28}$$

Similarly for the reflected portion of the light, R_{iso}, the portion of the ground that can be seen can be calculated:

$$R^0_{iso} = \frac{1}{2\pi} \int_{-\Sigma_C}^{0} \int_0^{\pi} sin^2(\phi) \cdot sin(\theta + \Sigma_C)\mathrm{d}\phi\mathrm{d}\theta = \frac{1 - \cos(\Sigma_C)}{2} \tag{29}$$

Near Shading

When modules are arranged in multiple rows, each row limits the share of the sky available to the row behind it; this limits the available diffuse light throughout the year. Using the angle from the collector to the top edge of the collector in front of it (γ), the formula becomes:

$$D^0_{iso} = \frac{1}{2\pi} \int_{\gamma}^{\pi-\Sigma_C} \int_0^{\pi} sin^2(\phi) \cdot sin(\theta + \Sigma_C)\mathrm{d}\phi\mathrm{d}\theta = \frac{1 + \cos(\Sigma_C + \gamma)}{2}$$

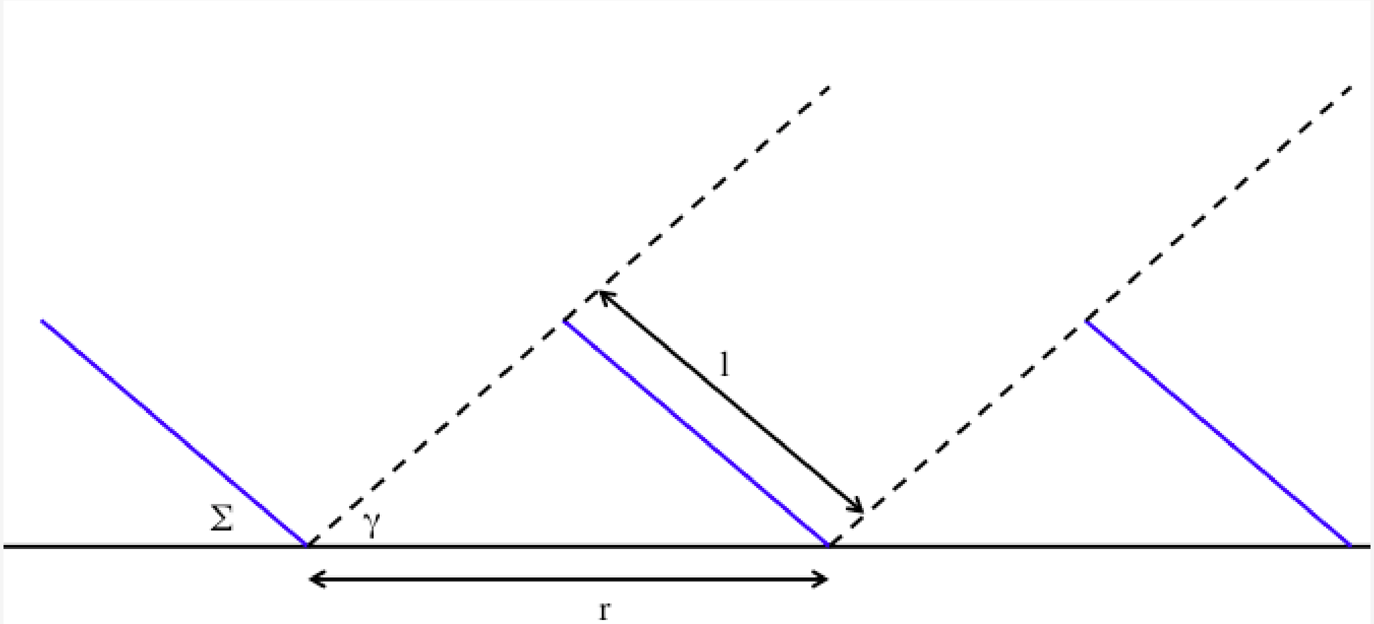


Diagram of Angles for Shed Calculations

This is used for any commercial or utility system with rows of racked modules, without specific geometric data. However, as a result of using detailed design data for the system, the exact module locations are known, which allows us to add back in additional irradiation for modules on the edge of the array:

$$D_{iso} = \frac{1+\cos(\Sigma_C+\gamma)}{2} + \frac{1}{2\pi}\int_0^{\gamma}\left(\int_0^{\phi_{west}} sin^2(\phi)\cdot sin(\theta+\Sigma_C)\mathrm{d}\phi + \int_{\phi_{east}}^{\pi} sin^2(\phi)\cdot sin(\theta+\Sigma_C)\mathrm{d}\phi\right)$$

Similarly, for the reflected factor, the only portion of the ground available for reflected light is the ground between the rows of panels. If the angle from the module to the ground beneath the top edge of the collector in front of it is ϵ, then the reflected radiation adjustment becomes:

$$R_{iso} = \frac{1}{2\pi}\int_{-\Sigma_C}^{\epsilon}\int_0^{\pi} sin^2(\phi)\cdot sin(\theta+\Sigma_C)\mathrm{d}\phi\mathrm{d}\theta$$

In practice this term is quite small compared to the other factors.

For use in output, given that the baseline isotropic adjustments (D^0_{iso} and R^0_{iso}) have already been incorporated into the transposition models, so adjusting diffuse and reflected irradiance only requires adjusting output by the relative difference:

$$I'_{DC} = I_{DC}\left(\frac{D_{iso}}{D^0_{iso}}\right)$$
$$I'_{RC} = I_{RC}\left(\frac{R_{iso}}{R^0_{iso}}\right)$$

Note that for beam irradiance, the near shading effect is simply that when the beam portion of the light is obstructed by any part of the preceding bank, all beam irradiance is lost – that is whenever the solar altitude angle is lower than the next bank of modules ($\beta_S <= \gamma$) and is between the bounds of the edges of the next bank of modules ($\phi_{east} <= \phi_S <= \phi_{west}$), the module is considered shaded and beam irradiance (I_{BC}) is set to 0.

Incidence Angle Adjustment

Because of reflection of incident light off the surface of the modules, all types of irradiance should be adjusted based on the incidence angle. This has been parameterized by ASHRAE as:

$$\rho(\theta_i) = 1 - b_0\left(\frac{1}{cos(\theta_i)} - 1\right) \tag{30}$$

where:

θ_i
The angle of incidence between the light and the surface of the collector
b_0
a constant, typically 0.05

Beam Reflection Calculation

For beam irradiance, this adjustment is straightforward, as can be seen equation(31), but for diffuse and albedo irradiance, this formula must be integrated across the visible portion of the sky (for diffuse) or the visible portion of the ground (for reflected).

$$I'_{BC} = \rho(\theta_i)I_{BC} \tag{31}$$

Diffuse and Reflected Calculation

For diffuse and albedo irradiance, the incidence angle modifier ($\rho(\theta_i)$ must be integrated across the portion of the sky available to the collector (just as in the isotropic adjustments) using equation (27). The specific formulation is given by:

$$\frac{1}{2\pi}\int\int sin^2(\phi)\cdot sin(\theta + \Sigma_C)\rho(\theta_i)\mathrm{d}\theta\prime\mathrm{d}\phi$$

This equation must be integrated numerically; evaluating the integral over the portion of the sky available to the collector (just as in the isotropic adjustment) it produces D_{iam}, the total adjustment for diffuse accounting for near shadings and the incidence angle adjustment. Similarly evaluating over the ground in front of the collector R_{iam}, the equivalent adjustment for the reflected/albedo irradiance. When doing this integral, the adjustments for shadings are taken into account by the bounds of the integral, so this calculation becomes the only adjustment necessary for diffuse and reflected irradiance after the initial transposition model.

$$I'_{DC} = I_{DC}\left(\frac{D_{iam}}{D^0_{iso}}\right) \tag{32}$$

$$I'_{RC} = I_{RC}\left(\frac{R_{iam}}{R^0_{iso}}\right) \tag{33}$$

Soiling Effects

The last adjustment made to irradiance is a user defined soiling adjustment. The user can define a percentage factor to reduce irradiance each month to describe soiling or other effects. This factor is applied to irradiance before any other calculations (e.g. module temperature, line loss, etc.) are performed.

Near Shading

Shading serves to reduce the irradiance on specific modules within the system based on local obstructions (e.g. trees, HVAC units). Once the geometry of these obstructions is loaded into the field, one can test whether direct irradiance from the sun must pass through any obstructions while calculating irradiance. If so, Beam Irradiance is reduced to zero while diffuse irradiance continues to drive module output.

At present the shading scene is evaluated from a top down perspective and tied to the location of each module based on its location in the horizontal plane. If any portion of the module is shaded, the whole module is considered shaded and loses beam irradiance ($I_{BC} = 0$)

One can load a shading scene into HelioScope using SketchUp – 3D software released with both a free and professional version. This software allows one to easily generate 3d-models tied to a specific location using a searchable, preexisting database of standard shapes (e.g. trees, buildings). These can then be scaled to specific dimensions (height, width and length) to ensure they approximate the actual obstructions and imported directly into a field using a custom plugin (currently in development).

Other Mismatch

In practice, there are many other sources of mismatch present in a solar array: clouds, manufacturing variance, module degradation, soiling, etc. These effects are not specifically modeled, but can be estimated on a case-by-case to simulate other field test-cases. For the time being this mismatch can be estimated at the scenario level using standard distributions for temperature, module binning and irradiance. Currently three parameters are provided:

Irradiation Variance
A standard deviation for module level variation in light levels across the field (assumes a normal distribution of light levels)
Temperature Spread
The total range of cell temperatures across the field, centered around the modeled cell temperature (pulled from a linear distribution)
Min/Max Module Tolerance
A linear range in which each modules out varies linearly (by adjusting short-circuit current)

Note that wherever randomized numbers are used, the same random seed is used in each scenario to prevent random variation from driving design choices.

Conclusions

To simulate the output for a test field, we calculate the available insolation, cell temperature and other desired environmental effects for a specific slice of time and then 'solve' the field; ensuring that every equation describing an electrical components input and output power is internally consistent. This calculation is then performed for each hour within a given year, allowing us to identify the total annual production.

Combining a granular, component-based model of a specific solar array with an environmental model allows us to simulate field output under a robust, yet flexible, range of environmental assumptions to develop a deeper understanding of the drivers of field production.